

LIMITS AND DERIVATIVES

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INTRO

We say $\lim_{x \rightarrow a^-} f(x)$ is the expected value of f at $x = a$ given the values of f near

x to the left of a . This value is called the left hand limit of f at a .

We say $\lim_{x \rightarrow a^+} f(x)$ is the expected value of f at $x = a$ given the values of

f near x to the right of a . This value is called the right hand limit of $f(x)$ at a .

If the right and left hand limits coincide, we call that common value as the limit

of $f(x)$ at $x = a$ and denote it by $\lim_{x \rightarrow a} f(x)$.

LHL AND RHL

Illustration 2 Consider the function $f(x) = x^3$. Let us try to find the limit of this function at $x = 1$. Proceeding as in the previous case, we tabulate the value of $f(x)$ at x near 1. This is given in the Table 13.5.

Table 13.5

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.729	0.970299	0.997002999	1.003003001	1.030301	1.331

From this table, we deduce that value of $f(x)$ at $x = 1$ should be greater than 0.997002999 and less than 1.003003001 assuming nothing dramatic happens between

$$\therefore \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x) = 1$$

1. A constant function takes the same value for all values of x , hence, limit will also be same
2. If value of lhl $\neq$ rhl, limit is not defined
3. However, at a given point the value of a function and its limit may differ, even when both are defined

Algebra of limits

Limit of sum of two functions is sum of the limits of the functions

$$\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

Limit of difference of two functions is difference of the limits of the functions

$$\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

Limit of product of two functions is product of the limits of the functions

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

Limit of quotient of two functions is quotient of the limits of the functions (whenever the denominator is non zero)

$$\lim_{x \rightarrow a} [f(x)/g(x)] = \lim_{x \rightarrow a} f(x) / \lim_{x \rightarrow a} g(x)$$

In particular as a special case of (iii), when g is a constant function such that $g(x) = \lambda$, for some real number λ , we have

$$\lim_{x \rightarrow a} [\lambda \cdot f(x)] = \lambda \cdot \lim_{x \rightarrow a} f(x)$$

Limits of polynomial functions

A function f is said to be a polynomial function if $f(x)$ is zero function or if $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, where a_i are real numbers such that $a_n \neq 0$ for some natural number n .

1. $\lim_{x \rightarrow a} x^n = a^n$
2. let $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ be a polynomial function. then, $f(x) = f(a)$