

Systems of Linear Equations - Methods

Elimination Methods	<i>Inverse Method</i>	Iterative Methods
Need scale system because system becomes more sensitive to round offs	solve multiple times for different constants	make unknowns the subject of equations
Maximum Coefficients on Main diagonal	Advantages	default all unknowns are 0
<i>Gauss Elimination</i>	calculate inverse once	Dominant Diagonal System DDS
1 forward elimination 2 back substitution	iterate for dynamic cases	DDS ensures convergence
eliminate what is below main diagonal	Limitations	<i>Gauss Seidel</i>
Issues	matrix has to have a solution	use updated values in equations
Zero at pivot - solution: switch rows	under-determined systems (# equations < # unknowns)	if system is converging
ill conditioned system - round off	do not have an inverse - infinite solutions	<i>Jacobi</i>
Limitations	Augmentation	update values at the end of each iteration
Lengthy- Cumbersome- Time consuming	$[A:I] \rightarrow [I:A]$	help overcome divergence
2 distinct steps	equations have to be linearly independent	Relaxation
<i>Gauss Jordan</i>		$X_{new} = X_{old} + (1 - X_{old})$
eliminate what is above and below the main diagonal		$0 < \sim < 2$
translate from coefficient matrix to identity matrix		$\sim = 0$ diverging (initial conditions are most accurate)

Systems of Linear Equations - Methods (cont)

Advantage: no need for back substitution	$\sim = 1$ regular
	$\sim = 2$ converging
	$\sim < 1$ diverging or converging with fluctuations
	$\sim > 1$ converging without fluctuations
	as system grows, $\sim$ is close to 1

Roots of Non linear Equations — Numerical Methods

Bracketed Methods	Open Methods
2 initial guesses bracket the root	initial guesses do not have to bracket root
to check that initial guesses bracket root: $f(x_l)f(x_u) < 0$	Newton Raphson
Bisection Method	Takes into account 1 initial guess 2 function behavior 3 rate of change
$X_m = (X_l + X_u) / 2$	$X_{i+1} = X_i - (f(x_i) / f'(x_i))$
Limitations:	pitfalls
1 miss roots	diverge due to inflection point
2 inefficient (time consuming)	converge to local min/max
3 if even # of roots between initial guesses are missed	jumping roots- converge to a different root
4 disregard function behavior; function of initial guesses	if x_i is close to zero, it will offshoot
False Position	Limitation: differentiation
$X_r = X_u - (f(x_u) * (x_l - x_u)) / (f(x_l) - f(x_u))$	Secant Method
in some cases, bisection may converge faster	$x_{i+1} = x_i - ((f(x_i) * (x_{i-1} - x_i)) / (f(x_{i-1}) - f(x_i)))$
	Modified Secant
	1 initial guess
	$x_{i-1} = x_i + o(x_i)$



Roots of Non-linear Equations

Analytical Solution	Graphical Solution
cannot solve complex equations	Visual Preceptions
Roots of an equation	Miss roots due to choice of window
find the value of independent variable when the dependent variable is zero.	

Systems of Linear Equations

Graphical Solution	# equations = # unknowns
Visual perception - accuracy	1 solution
Time consuming	# equations < # unknowns
impractical beyond 3D	infinite solutions
	# equations > # unknowns
	1 solution (redundant equation)
	no solution - do not intersect

Systems of Linear Equations - Cranmer's Rule

D = determinant of coefficients	Limitations
Dn = determinant of coefficients with n column replaced with B matrix	Time consuming
Singular System D=0	if D=0
1 no solution	ill- conditioned system
2 infinite solutions	D is close to 0
	instruction is a region
	sensitive to round offs

