

VECTORS

2 vectors equal $\leftrightarrow$ same magnitude + same direction

norm/magnitude/length of vector $\|v\| = \sqrt{v_1^2 + \dots + v_n^2}$

unit vector $\rightarrow \|v\| = 1$. ($v / \|v\|$)

$\|cv\| = |c| \|v\|$

distance $d(u,v) = \|u-v\|$

Dot product $u \cdot v = (u_1 v_1 + \dots + u_n v_n)$

$n \cos(\theta) = u \cdot v / (\|u\| \|v\|)$

u & v are orthogonal when $\text{dot}(u,v) = u \cdot v = 0$

Equations of Lines/Planes

vector form $x = x_0 + tv$

parametric in R^2 $x = x_0 + ta, y = y_0 + tb$

parametric in R^3 $x = x_0 + ta, y = y_0 + tb, z = z_0 + tc$

General form of plane R^3 $Ax + By + Cz = D$

point normal eq of a plane $n \cdot (x - x_0)$

vector eq of a plane $x = x_0 + sv_1 + tv_2$

Inverse Matrices

Inversion Algorithm

$[A | I] \rightarrow \dots [I] \dots \rightarrow [I | A^{-1}]$

Invertible Matrix Theorem

Invertible Matrix Theorem

Invertible Matrix Theorem (cont)

If A is an $n \times n$ matrix, and if T_A is the linear operator on R^n with standard matrix A , then the following statements are equivalent.

1. A is invertible.
2. The RREF of A is Identity matrix.
3. A can be written as a product of elementary matrices.
4. $Ax = 0$ has only the trivial solution: $x = 0$
5. $Ax = b$ has exactly one solution for every vector b in R^n : $x = A^{-1}b$.
6. $Ax = b$ is consistent for every vector b in R^n .
7. The column vectors of A are linearly independent.
8. The row vectors of A are linearly independent.
9. $\det(a) \neq 0$.
10. $\lambda = 0$ is not an eigenvalue of A .
11. T_A is one-to-one.
12. T_A is onto.
13. The column vectors of A span R^n .
14. The row vectors of A span R^n .
15. The column vectors of A form a basis for R^n .
16. The row vectors of A form a basis for R^n .
17. $\text{rank}(A) = n$
18. $\text{nullity}(A) = 0$
19. A has full column rank.

EigenSTUFF

The scalar λ is called an Eigenvalue of A when there is a nonzero vector x such that $Ax = \lambda x$.

Vector x is an Eigenvector of A corresponding to λ .

The set of all eigenvectors with the zero vector is a subspace of R^n called the eigenspace of λ .

1. Find Eigenvalues: $\det(\lambda I - A) = 0$
2. Find Eigenvectors: $(\lambda I - A)x = 0$

If A is a triangular matrix then its eigenvalues are on its main diagonal

