

Derivatives

- Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$
- Product Rule: $(uv)' = u'v + uv'$
- Quotient Rule: $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
- Chain Rule: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Trigonometric Identities

- Pythagorean Identities: $\sin^2(x) + \cos^2(x) = 1$
- Sum and Difference Identities: $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
- Double Angle Identities: $\sin(2x) = 2\sin(x)\cos(x)$

Logarithmic Properties

- Logarithmic Properties:**
- Logarithmic properties are rules that govern the behavior of logarithmic functions. Some key properties include:
 - Product Rule: $\log_b(xy) = \log_b(x) + \log_b(y)$
 - Quotient Rule: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
 - Power Rule: $\log_b(x^n) = n\log_b(x)$

Approximating Area Under a Curve

- Left, Right, Midpoint Approximations:
 - Left Sum = $\sum_{i=1}^n f(x_{i-1})\Delta x$
 - Right Sum = $\sum_{i=1}^n f(x_i)\Delta x$
 - Midpoint Sum = $\sum_{i=1}^n f(\bar{x}_i)\Delta x$, where $\bar{x}_i$ is the midpoint of the i th interval.
- Trapezoidal Rule:
 - Trapezoidal Rule = $\frac{\Delta x}{2}[f(a) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(b)]$
- Simpson's Rule:
 - Simpson's Rule = $\frac{\Delta x}{3}[f(a) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(b)]$

Power Sum Rules

$$\begin{aligned}\sum_{i=1}^n i &= \frac{n(n+1)}{2} \\ \sum_{i=1}^n i^2 &= \frac{n(n+1)(2n+1)}{6} \\ \sum_{i=1}^n i^3 &= \left(\frac{n(n+1)}{2}\right)^2\end{aligned}$$

Definite Integrals

- **Definition:** $\int_a^b f(x)dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_{i-1})\Delta x$.
- **Properties:** Flip bounds (negate), break into smaller integrals, comparison test.
- **Calc 1 Technique:** Fundamental understanding of integration and its geometric interpretation as area under a curve.

u-Substitution

- Replace x with $u(x)$, convert bounds, $dx = \frac{du}{u'(x)}$.
- **Calc 1 Technique:** Use when an integral's integrand is a composite function, resembling the chain rule's application in reverse.

Integration by Parts

- $\int u dv = uv - \int v du$
- LIATE Rule: Choose u as the algebraic function that appears first in the list of functions: Logarithmic, Inverse trigonometric, Algebraic, Trigonometric, Exponential.

Integrals of Trigonometric Functions

- Integrals of Basic Trigonometric Functions:**
- $\int \sin(x) dx = -\cos(x) + C$
 - $\int \cos(x) dx = \sin(x) + C$
 - $\int \tan(x) dx = -\ln|\cos(x)| + C$
- Integrals Involving Secant and Cosecant:**
- $\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C$
 - $\int \csc(x) dx = -\ln|\csc(x) + \cot(x)| + C$
- Integrals Involving Products of Trigonometric Functions:**
- $\int \sin^n(x) \cos^n(x) dx$ can often be solved using trigonometric identities or by applying reduction formulas for powers of sine and cosine.
- Trigonometric Substitution:**
- Used to solve integrals involving radicals or quadratic expressions by substituting trigonometric functions for variables.
 - For example, if $\sqrt{a^2 - x^2} dx$ appears, you could let $x = a \sin(\theta)$ or $x = a \cos(\theta)$ to simplify the integral.

Hyperbolic Trig Functions

- Hyperbolic functions are analogs of circular trigonometric functions, but they are based on hyperbolas instead of circles. The primary hyperbolic functions are:
 - Hyperbolic Sine: $\sinh(x) = \frac{e^x - e^{-x}}{2}$
 - Hyperbolic Cosine: $\cosh(x) = \frac{e^x + e^{-x}}{2}$
 - Hyperbolic Tangent: $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$
- Hyperbolic functions have properties similar to trigonometric functions, such as even/odd symmetry and relationships between their derivatives and themselves.

Power Series

$$\sum_{n=0}^{\infty} a_n (x - a)^n \text{ with a radius of convergence } R.$$

Order of Convergence

$$\lim_{n \rightarrow \infty} \frac{|a_{n+1} - L|}{|a_n - L|^p} = C, \text{ where } C \text{ is a constant and } p \text{ is the order of convergence.}$$

Convergence

- Convergence and Divergence:**
- A series is said to converge if the sequence of partial sums approaches a finite limit as the number of terms increases indefinitely. If the limit exists, it is called the sum of the series.
 - Conversely, a series diverges if the sequence of partial sums does not approach a finite limit. In this case, the series does not have a sum.
 - Example of convergent series: $\sum_{n=0}^{\infty} \frac{1}{2^n} = 1$, known as the geometric series.
 - Example of divergent series: $\sum_{n=1}^{\infty} n$ diverges, as the partial sums increase without bound.
- Convergence Tests:**
- Convergence tests are used to determine whether a series converges or diverges. Some common convergence tests include:
 - Divergence Test: If $\lim_{n \rightarrow \infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
 - Geometric Series Test: $\sum_{n=1}^{\infty} ar^{n-1}$ converges to $\frac{a}{1-r}$ if $|r| < 1$.
 - Integral Test: If $f(x)$ is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x) dx$ either both converge or both diverge.

Types of Convergence:

- Absolute Convergence: A series $\sum_{n=1}^{\infty} |a_n|$ converges.
- Conditional Convergence: A series $\sum_{n=1}^{\infty} a_n$ converges, but $\sum_{n=1}^{\infty} |a_n|$ diverges.

Sequences and Series

Sequences:

- A sequence is an ordered list of numbers written in a specific order. It can be finite or infinite.
- Formally, a sequence is denoted as (a_n) or $\{a_n\}$, where a_n represents the n th term of the sequence.
- Example: $1, 1/2, 1/3, 1/4, \dots$ is a sequence with the formula $a_n = \frac{1}{n}$.

Series:

- A series is the sum of the terms of a sequence. It represents the total when an infinite number of terms are added together.
- Formally, a series is denoted as $\sum_{n=1}^{\infty} a_n$ or $a_1 + a_2 + a_3 + \dots$
- Example: $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ represents the series of the sequence mentioned earlier.

Partial Fractions

- Partial fraction decomposition is a technique used to break down a rational function into simpler fractions.
- The general process involves:
 - 2.1. Factorizing the denominator into linear and irreducible quadratic factors.
 - 2.2. Writing the partial fraction decomposition with undetermined coefficients for each factor.
 - 2.3. Determining the values of the undetermined coefficients by equating coefficients.
- Example: $\frac{3x^2 + 5x + 2}{x^2 - 2x + 2}$ can be decomposed into partial fractions as $\frac{A}{x} + \frac{B}{x-1} + \frac{C}{x^2-2x+2}$.

Logistical Growth Function

$$P(t) = \frac{K}{1 + e^{-\frac{r}{K}t}}$$

r = growth rate, K = carrying capacity

Improper Integrals

$$\text{• Example: } \int_1^{\infty} \frac{1}{x^p} dx \text{ converges if } p > 1 \text{ and diverges if } p \leq 1.$$

Integrals where the limits of integration are infinite or where the function being integrated is undefined at certain points within the interval.

Taylor Polynomials

- A Taylor polynomial of degree n for a function $f(x)$ centered at a point $x = a$ is a polynomial that approximates $f(x)$ near $x = a$.
- The general form of a Taylor polynomial is:
$$P_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$
- Here, $f^{(n)}(a)$ denotes the n th derivative of $f(x)$ evaluated at $x = a$, and $n!$ represents the factorial of n .

Taylor's Theorem

- Taylor's Theorem provides a way to quantify the error between a function $f(x)$ and its Taylor polynomial approximation $P_n(x)$.
- It states that if $f(x)$ is $(n+1)$ -times differentiable on an interval containing $x = a$, then there exists a number c between a and x such that the error $R_n(x) = f(x) - P_n(x)$ can be expressed as:
$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1}$$
- This error term is known as the remainder or residual term of the Taylor polynomial.

Maclaurin Series

- A Maclaurin series is a special case of a Taylor series where the polynomial is centered at $x = 0$ (i.e., $a = 0$).
- The Maclaurin series expansion of a function $f(x)$ is given by:
$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots$$
- Essentially, the Maclaurin series represents the Taylor series expansion of a function around the point $x = 0$.

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