

Set symbols			
Set	$\{ \}$	a collection of elements	$A = \{3, 7, 9, 14\},$ $B = \{9, 14, 28\}$
Such that	$ $	such that	$A = \{x \mid x \in \mathbb{N}, x < 0\}$
Intersection	$A \cap B$	belongs to both A AND B at	$A \cap B = \{9, 14\}$
Union	$A \cup B$	belongs to either A OR B	$A \cup B = \{3, 7, 9, 14, 28\}$
Subset	$A \subseteq B$	A is a subset of B. Set A is included in set B.	$\{9, 14, 28\} \subseteq \{9, 14, 28\}$
Proper subset / strict subset	$A \subset B$	A is a subset of B, but A is not equal to B	$\{9, 14\} \subset \{9, 14, 28\}$
Not a subset	$A \not\subseteq B$	Set A is not a subset of set B	$\{9, 66\} \not\subseteq \{9, 14, 28\}$
Superset	$A \supseteq B$	A is a superset of B. Set A includes set B	$\{9, 14, 28\} \supseteq \{9, 14, 28\}$
Proper superset / strict superset	$A \supset B$	A is a superset of B, but B is not equal to A	$\{9, 14, 28\} \supset \{9, 14\}$
Not superset	$A \not\supseteq B$	Set A is not a superset of set B	$\{9, 14, 28\} \not\supseteq \{9, 66\}$
Power set	2^A	All subsets of A	
Power set	$P(A)$	All subsets of A	

Set symbols (cont)			
Equality	$A=B$	Both sets have the same members	$A = \{3, 9, 14\},$ $B = \{3, 9, 14\},$ $A=B$
Complement	A^c	All the objects that do not belong to set A	
Relative complement	$A \setminus B$ or $A - B$	Objects that belong to A and not to B	$A = \{3, 9, 14\},$ $B = \{1, 2, 3\},$ $A \setminus B = \{9, 14\}$
Symmetric difference	$A \Delta B$ or $A \oplus B$	Objects that belong to A or B but not to their intersection	$A = \{3, 9, 14\},$ $B = \{1, 2, 3\},$ $A \Delta B = \{1, 2, 9, 14\}$
Element of	$a \in A$	Set membership	$A = \{3, 9, 14\},$ $3 \in A$
Not element of	$x \notin A$	No set membership	$A = \{3, 9, 14\},$ $1 \notin A$
Ordered pair	(a, b)	Collection of 2 elements	
Cartesian product	$A \times B$	Set of all ordered pairs from A and B	
cardinality	$ A $ or $\#A$	The number of elements of set A	$A = \{3, 9, 14\},$ $ A =3$

Set operations	
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Commutativity	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Complementation	$A \cup \text{Not}(A) = U$ $A \cap \text{Not}(A) = \emptyset$
Idempotence	$A \cup A = A$ et $A \cap A = A$
Identité	$A \cup \emptyset = A$ $A \cap U = A$

Set operations (cont)	
Extrémité	$A \cup U = U$ $A \cap \emptyset = \emptyset$
Involution	$\text{Not}(\text{Not}(A)) = A$
Morgan's law	$(\text{Not}(A \cup B)) = \text{Not}(A) \cap \text{Not}(B)$ $(\text{Not}(A \cap B)) = \text{Not}(A) \cup \text{Not}(B)$
Distributivité	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Languages - Definitions	
Formal language	vocabulary + grammar rules
Monoid	Set having an internal binary operation $(+, -, *, \cup, \dots)$ and having a neutral element ϵ . Ex: $\langle E, +, \epsilon \rangle$ means all strings of E will be concatenated with operator +
Grammar	(V_n, V_t, P, S) when: - V_n is a finite non-empty set whose elements are variables - V_t is a finite non-empty set of terminal states - $V_n \cap V_t = \emptyset$ - P is a finite set whose elements are $\alpha \rightarrow \beta$, known as production rules when $\alpha, \beta, \in (V_n \cup V_t)^*$ but α should contain at least 1 symbol from V_n - S is a start symbol, where $S \in V_n$
Symbol	Element of a set. Ex for set $A = \{1, 2, 3\}$, 2 is an element
Alphabet	A finite and non empty set of symbols



Languages - Definitions (cont)

Length of a string Number of symbols in a string.
Ex: for $w = aabbc$, $|w| = 5$

Empty language $\emptyset$ contains no string

Empty string ϵ where $|\epsilon| = 0$

A^n Word of length n from the alphabet A

A^* Words of finite length from the alphabet A or null (can be empty: ϵ)

A^+ Words of finite length from the alphabet A and NOT NULL (no ϵ)

Regular expression Recursive language, the rules of the expression must be independent. Ex:
 $L1 = \{a^n b^{2n} \mid n \geq 0, m \geq 0\}$ and
 $L2 = \{a^n b^m \mid n = 2m\}$
Here $L1$ is regular and $L2$ is not. Because in $L2$ there is a dependency between n and m

Maths set reminders

N $\{0, 1, 2, 3, \dots\}$

Z $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

D $\{d \mid d \text{ is a nb. having a finite number of decimals}\}$

Q $\{r \mid r \text{ is a rational nb. that can be written as the quotient } a/b \text{ of a real integer number } a \text{ by a whole integer (not null) } b\}$

IR $\{\dots, -3, -2, -1, 0, \sqrt{2}, 1, \sqrt{2}, 2, e, 3, \pi, \dots\}$

Successive inclusions $N \subset Z \subset D \subset Q \subset IR$

Automata definitions

DFA Deterministic Finite Automata. A DFA has a deterministic because from each state we are able to determine the next state.

NDA Non Deterministic Finite Automata. A NDA is non deterministic because we can't always determine which will be the next state from the present state.

Operations on Languages

Union $L \cup M = \{x \mid x \in L \text{ or } x \in M\}$

Intersection $L \cap M = \{x \mid x \in L \text{ and } x \in M\}$

Difference (exclusion) $L \setminus M = \{x \mid x \in L \text{ and } x \notin M\}$

Complement on A^* $\text{Comp}(L) = A \setminus L = \{x \mid x \in A \text{ and } x \notin L\}$

$L \cup \emptyset = L$

$L \cap \emptyset = \emptyset$

Identity of Regular Expressions

$\emptyset + R = R$

$\emptyset \cdot R = R \cdot \emptyset = \emptyset$

$\Lambda \cdot R = R \cdot \Lambda = R$

$\Lambda^* = \Lambda$ and $\emptyset^* = \Lambda$

$R + R = R$

$R^* R^* = R^*$

$R \cdot R^* = R^* \cdot R$

$(R^*)^* = R^*$

$\Lambda + R \cdot R^* = R^* = \Lambda + R^* \cdot R$

$(P \cdot Q)^* P = P (Q \cdot P)^*$

$(P + Q)^* = (P^* \cdot Q^*)^* = (P^* + Q^*)^*$

$(P + Q) \cdot R = P \cdot R + Q \cdot R$ and $R(P + Q) = R \cdot P + R \cdot Q$

Graph types

Reflexif All states have a loop, meaning $\delta(\text{state1}, \text{input}) = \text{state1}$

Symetric All states have a direct way back, meaning:
 $\delta(A, x) = B$ and $\delta(B, y) = A$

Graph types (cont)

Anti symmetric There never is a direct way back to the same state meaning, if:
 $\delta(A, x) = B$ then $\delta(B, y) \neq A$

Transitive There are shortcuts to a state, meaning if:

Grammar construction

Problem – Suppose :

$L(Gr) = \{a^n b^m c^k \mid n \geq 0, k > 1, m = n+k\}$

We have to find out the grammar Gr which produces $L(Gr)$.

Solution:

Possible word $w = a^4 b^9 c^5$

We need to decompose b^9 : $w = a^4 b^4 b^5 c^5$

Now we can define :

- $S1$ having the same number of a followed by the same number of b .

- $S2$ having the same number of b followed by the same number of a .

So:

$S = S1 S2$

$S1 = a S1 b \mid \Lambda$

$S2 = b S2 a \mid bbcc$

Why $bbcc$ as alternative to $S2$? Because in the case of a minimum word $w = b^2 c^2 = bbcc$ because $k > 1$ so in this case we do have $m = n+k = 0+2$